Calculation of a Three-Dimensional Locally Elliptic Flow with a Zonal Equation Method

Matthew J. Warfield* and B. Lakshminarayana†
Pennsylvania State University, University Park, Pennsylvania 16802

Abstract

THE Navier-Stokes equatons are elliptic or time hyperbolic, but parabolizing assumptions can be made under certain conditions leading to various boundary-layer and space-marching formulations that can greatly accelerate the solution time. A zonal equation method is presented which uses zones where ellipticity is high for solution by Navier-Stokes methods, and zones where parabolizing assumptions are valid for solution by space-marching techniques. The concept of localized ellipticity is explored and evaluated. The zonal equation method algorithm is outlined and results of its application presented. Good results are shown with significant savings in time over a full Navier-Stokes solution.

Contents

Pressure gradient and streamwise diffusion terms must be deleted or treated specially in reducing full Navier-Stokes equations to equations amenable to space marching. These terms are important in characterizing the ellipticity present in each flow. Quantifying the ellipticity in a flow should be addressed prior to the implementation of a zonal equation method. An ellipticity parameter ϵ can be established by using ratios of some of the ellipticity producing terms in the Navier-Stokes equations. In a three-dimensional flow, ellipticity can be characterized by:

$$\epsilon_1^3 = \left(\frac{\partial p}{\partial y}\frac{\partial^2 u}{\partial x^2}\right) \Bigg/ \left(\frac{\partial p}{\partial x}\frac{\partial^2 u}{\partial y^2}\right) \qquad \epsilon_2^3 = \left(\frac{\partial p}{\partial z}\frac{\partial^2 u}{\partial x^2}\right) \Bigg/ \left(\frac{\partial p}{\partial x}\frac{\partial^2 u}{\partial z^2}\right) \qquad (1)$$

where x and u are in the streamwise direction, and secondary velocity derivatives have been ignored. Here, ϵ_1 and ϵ_2 clearly go to zero in a purely parabolic flow such as a fully developed straight duct flow. Testing of this ellipticity parameter in a benchmark flow yields information on its distribution. Figure 1 shows a test case consisting of a straight duct with a square post constriction. This case contains significant ellipticity in the form of complex pressure gradients, flow separation, and strong crossflow while still containing the parabolic characteristics of straight duct flow at large distances from the post. A benchmark solution was obtained with a full Navier-Stokes solution to develop the ellipticity concept and to facilitate comparison with zonal method results.

The ellipticity in the present case is presented in Fig. 2. The strong ellipticity in the immediate vicinity of the post on all sides is evident, but there is less well-defined ellipticity at a distance from the top of the post. The most important information obtained from Fig. 2 is the actual 0(1) values of ellipticity in locations where ellipticity would be expected to be high and also the very small values obtained at a distance from the post.

The zonal equation method algorithm is shown in Fig. 3. The Navier-Stokes solution was obtained by time marching using the method of pseudocompressibility.[1] The space-marching solution is with the method of Pauagare and Lakshminarayana.[2] The assumed pressure field and ellipticity are derived from the initial partial convergence time-marching solution. This is an important feature of the present technique: the pressure interaction between the parabolic and elliptic regions.

The test case was solved with three zones: Parabolic upstream to $x = -0.6$, elliptic for $-0.6 < x < 1.5$, and again parabolic for $x > 1.5$. The solution was compared to the benchmark Navier-Stokes solution. Figure 4 shows comparisons of streamwise velocity along the symmetry plane at several locations for the post case. In all cases there is quite good agreement between the zonal method solution and the benchmark solution, even in the stagnation region just ahead of the post. As would be expected, the comparison is nearly identical at a distance from the post. A reduction in time of approximately 60% was achieved with the zonal solution, which demonstrates the significant potential for computational savings with zonal equation methods.

Further details and applications of the zonal equation method can be found in Warfield and Lakshminarayana.[3]

Acknowledgments

This work was sponsored by the David W. Taylor Naval Ship Research and Development Center under a Naval Sea Systems Command contract to the Applied Research Laboratory, The Pennsylvania State University. The NASA Ames Research Center and Kevin R. Kirtley provided the foundation computer codes.

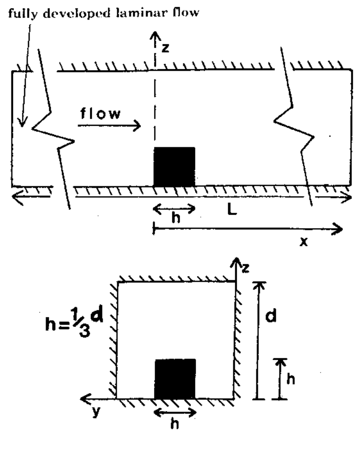

Fig. 1 Schematic of a straight duct with a square post.

Received May 23, 1987; presented as Paper 87-1141 at the AIAA Computational Fluid Dynamics Conference, June 9–11, 1987; synoptic received Jan. 18, 1990. Full paper available from AIAA Library, 555 W. 57th Street, New York, NY 10019. Price: microfiche, $4.00; hard copy, $9.00.

*NASA CFD Fellow, currently Senior Specialist Engineer, Boeing Defense and Space Group, Seattle, Washington. Member AIAA.

†Evan Pugh Professor, Applied Research Laboratory and Department of Aerospace Engineering. Fellow AIAA.

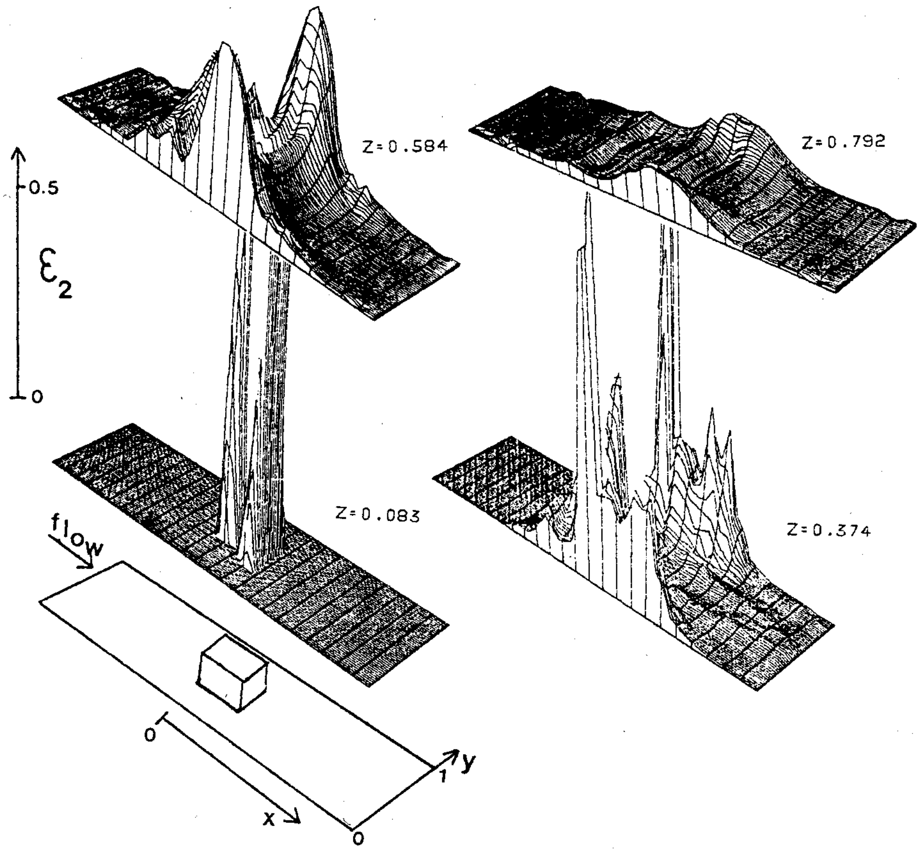

Fig. 2 Ellipticity distribution with height for a straight duct with a post.

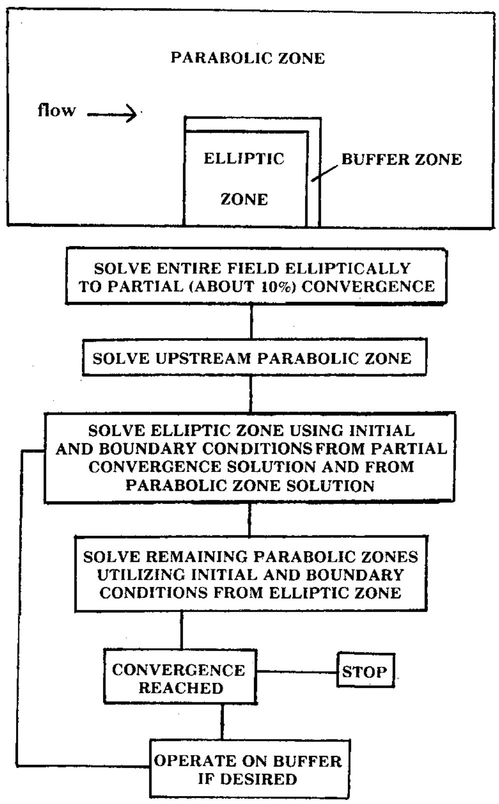

Fig. 3 The zonal equation algorithm for a two-dimensional cut in the computational domain.

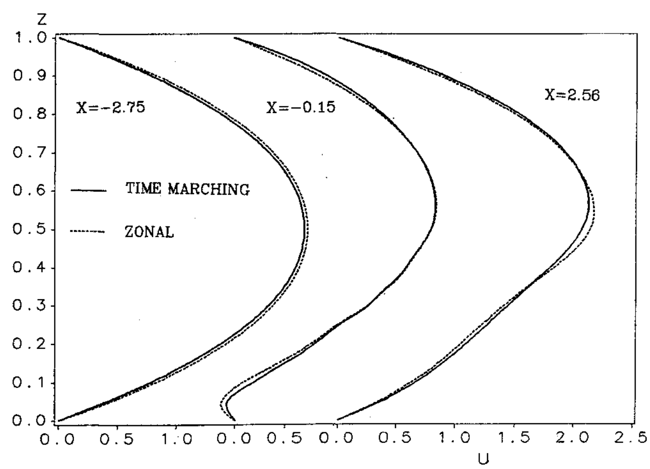

Fig. 4 Streamwise velocity comparisons along the symmetry line for a straight duct with a post.

References

[1]Kwak, D., Chang, J. L. C., Shanks, S. P., and Chakravarthy, S. R., "Three-Dimensional Incompressible Navier-Stokes Flow Solver Using Primitive Variables," *AIAA Journal,* Vol. 24, No. 3, 1986, pp. 390–396.

[2]Pauagare, M. P., Lakshminarayana, B., "A Space Marching Method for Viscous Incompressible Internal Flows," *Journal of Computational Physics*, Vol. 64, No. 2, 1980, pp. 389–415.

[3]Warfield, M. J., Lakshminarayana, B., "Calculation of Three Dimensional Locally Elliptic Flow with a Zonal Equation Method," AIAA Paper 87-1141, 1987.